(13)

AD-A144 057

NEW METHOD IN ELEMENTARY PARTICLE DETECTION

Final Report

Contract F49620-83-C-0095

ARPA Order 4099

Program Code 3A 10

DTIC
ELECTE
S   AUG 9 1984   D
D

UNIVERSITY OF MARYLAND
DEPARTMENT OF PHYSICS AND ASTRONOMY
COLLEGE PARK, MARYLAND

84 08 07 124

REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER AFOSR-TR- 84-0619	2. GOVT ACCESSION NO. A144 057	3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle) NEW METHOD IN ELEMENTARY PARTICLE DETECTION	5. TYPE OF REPORT & PERIOD COVERED Final Report
	6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s) J. Weber	8. CONTRACT OR GRANT NUMBER(s) F-49620-83-C-0095

9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Physics and Astronomy University of Maryland College Park, Maryland 20742	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS ARPA Order 4099 Program Code 3A 10

11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research Bolling Air Force Base Washington, D.C. 20332	12. REPORT DATE June 30, 1984
	13. NUMBER OF PAGES 42

14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Defense Advanced Research Projects Agency 1400 Wilson Boulevard Arlington, Virginia 22209	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Coherent Scattering, Elementary Particle Detection

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Theory is given for momentum transfer to an ensemble of particles by incident neutrinos or antineutrinos, in such a way that subsequent measurements cannot reveal the detailed characteristics of this transfer. It is shown that large scattering cross sections may be obtained, proportional to the square of the number of scatterers.

Experiments are described for observations at energies of about 12 kilovolts and in the one million electron volt region.

DD FORM 1473 EDITION OF 1 NOV 65 IS OBSOLETE
1 JAN 73

ABSTRACT

Theory is given for momentum transfer to an ensemble of particles by incident neutrinos or antineutrinos, in such a way that subsequent measurements cannot reveal the detailed characteristics of this transfer. It is shown that large scattering cross sections may be obtained, proportional to the square of the number of scatterers.

Experiments are described for observations at energies of about 12 kilovolts and in the one million electron volt region.

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)
NOTICE OF TRANSMITTAL TO DTIC
This technical report has been reviewed and is
approved for public release IAW AFR 190-12.
Distribution is unlimited.
MATTHEW J. KERPER
Chief, Technical Information Division

Introduction

Large numbers of interacting particles and long observation times have been required for weak interaction experiments at low energies. Total cross sections are proportional to the number of scatterers.

For the scattering of electromagnetic waves by macroscopic quantities of matter, the total cross sections in the x ray region are also proportional to the number of scatterers.[1,2] However for wavelengths large in comparison with dimensions of a macroscopic volume of scatterers, the total cross section may be proportional to the square[3] of the number of scatterers.

Research reported here explores a new method for obtaining weak interaction cross sections proportional to the square of the number of scatterers. In order to understand how this might be accomplished, we consider first the non relativistic theory of scattering by a two dimensional array of scattering potentials.

Scattering by a Planar Array

Let us imagine that there are N scatterers equally spaced along the x and y axes, (Figure 1). The x and y scatterer spacing is b units. A beam of particles has incident momentum $\bar{P}_{IO}$ and momentum $\bar{P}_{IF}$ after elastic scattering. The interactions occur in a volume V. Incident and scattered particles are represented by the wavefunctions

$$\psi_o = \frac{1}{\sqrt{V}} e^{i\bar{P}_{IO}\cdot\bar{R}/\hbar \, -iEt/\hbar} \quad , \quad \psi_F = \frac{1}{\sqrt{V}} e^{i\bar{P}_{IF}\cdot\bar{R}/\hbar \, -iEt/\hbar} \tag{1}$$
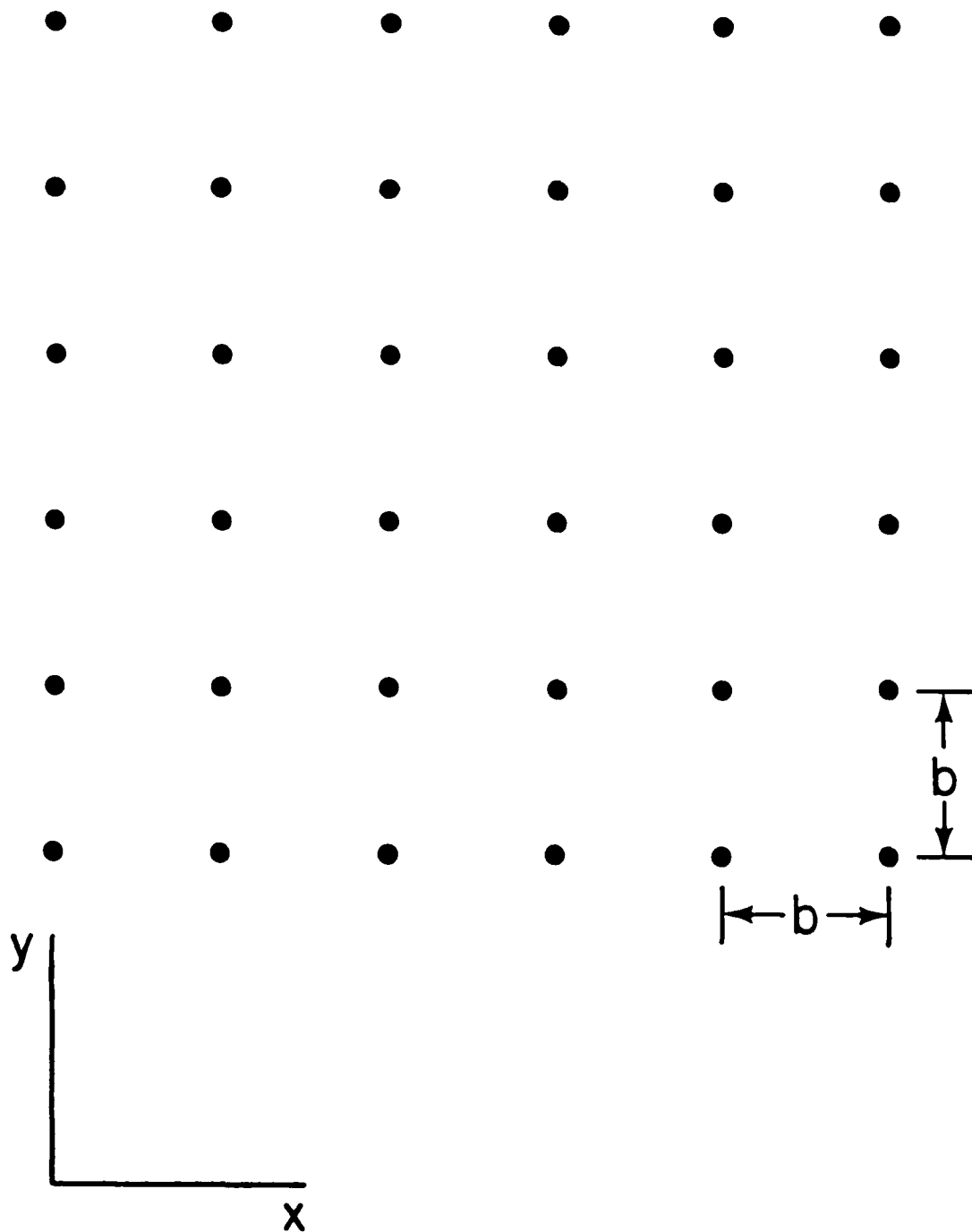
respectively.

1

Figure 1

Let the scattering potential be $U(\bar{r})$. The interaction matrix element is then

$$H' = \frac{1}{V} \int e^{-i\bar{P}_{IF}\cdot\bar{r}/\hbar} \, U(\bar{r}) \, e^{i\bar{P}_{IO}\cdot\bar{r}/\hbar} \, d^3x \qquad (2)$$

Suppose that each scatterer interacts via a delta function potential with integrated value B. Then $U(\bar{r})$ is given by

$$U(\bar{r}) = B \sum_{n_x=1}^{n_x=N^{1/2}} \sum_{m_y=1}^{m_y=N^{1/2}} \delta(x - n_x b) \, \delta(y - m_y b) \, \delta(z) \qquad (3)$$

For (3) H' is evaluated as

$$H' = \frac{B}{V} \sum_{m_y=1}^{m_y=N^{1/2}} \sum_{n_x=1}^{n_x=N^{1/2}} e^{i(\bar{P}_{IO}-\bar{P}_{IF})_x \, n_x b/\hbar} \, e^{i(\bar{P}_{IO}-\bar{P}_{IF})_y \, m_y b/\hbar} \qquad (4)$$

In (4) $(\bar{P}_{IO}-\bar{P}_{IF})_x$ and $(\bar{P}_{IO}-\bar{P}_{IF})_Y$ are the x and y components of $\bar{P}_{IO}-\bar{P}_{IF}$, respectively.

Fermi's golden rule gives a transition probability W with

$$W = \frac{2\pi}{\hbar} |H'|^2 \rho(E) \qquad (5)$$

2

The density of states $\rho(E)$ is computed by noting that in a range dE the total number of states for the outgoing particles is, for solid angle $d\Omega$

$$\rho(E)\,dE = \frac{V}{(2\pi\hbar)^3}\,|p_{IF}|^2\,dp_{IF}\,d\Omega \qquad (6)$$

For zero rest mass particles, $dE = c\,dp$

Expression (6) then gives

$$\rho(E) = \frac{V}{c(2\pi\hbar)^3}|p_{IF}|^2\,d\Omega \qquad (7)$$

The incident particle velocity c and normalization imply an incident particle flux

$$\frac{c}{V} \qquad (8)$$

The interaction matrix element (4) is the product of two geometric progressions which are readily summed. The scattering cross section σ is the quotient of (5) and (8), with

$$\sigma = \frac{|p_{ed}|^2 B^2}{4\pi^2\hbar^4 c^2}\int \frac{\sin^2\left[\frac{1}{2}N^{1/2}b\,(p_{Io}-p_{IF})_x/\hbar\right]\sin^2\left[\frac{1}{2}N^{1/2}b\,(p_{Io}-p_{IF})_Y/\hbar\right]}{\sin^2\left[\frac{1}{2}b(p_{Io}-p_{IF})_x/\hbar\right]\sin^2\left[\frac{1}{2}b(p_{Io}-p_{IF})_Y/\hbar\right]}\,d\Omega \qquad (9)$$

The differential cross section in (9) has a maximum value proportional

3

to N^2, given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{MAX} = \frac{|P_{Io}|^2 B^2 N^2}{4\pi^2 \hbar^4 c^2} \qquad (10)$$

For $\bar{P}_{Io}$ in a direction normal to the array, in the z direction, (10) corresponds to forward scattering with $\bar{P}_{IF}$ also in the z direction. For the forward scattering peak the solid angle $d\Omega$ is determined by the first zeros of the integrand of (9). These occur for

$$\left(\delta_{P_X}\right)_{\substack{NUMERATOR \\ ZERO}} = \left|\left(\bar{p}_{Io} - \bar{p}_{IF}\right)_X\right| = \frac{2\pi\hbar}{N^{1/2}b} \qquad (11)$$

$$\left(\delta_{P_Y}\right)_{\substack{NUMERATOR \\ ZERO}} = \left|\left(\bar{p}_{Io} - \bar{p}_{IF}\right)_Y\right| = \frac{2\pi\hbar}{N^{1/2}b}$$

(11) gives a solid angle

$$d\Omega \approx \pi \left|\frac{2\pi\hbar}{N^{1/2}b\, p_{Io}}\right|^2 \qquad (12)$$

The total cross section associated with this forward scattering peak is the product of (10) and (12), $\Delta\sigma_F$, given by

$$\Delta\sigma_F = \frac{\pi B^2 N}{\hbar^2 b^2 c^2} \qquad (13)$$

4

A study of (9) indicates that there are other peaks in addition to the forward peak.

There will be a peak for each value of $\bar{P}_{Io} - \bar{P}_{IF}$ which gives a zero in the denominator of (9). These occur at intervals defined by

$$\left(\delta_{p_x}\right)_{\substack{DENOMINATOR \\ ZERO}} = \frac{2\pi\hbar}{b} \tag{14}$$

$$\left(\delta_{p_Y}\right)_{\substack{DENOMINATOR \\ ZERO}} = \frac{2\pi\hbar}{b}$$

The total number of peaks n_p is the number of cells of area $\left(\delta_{p_x}\delta_{p_Y}\right)_{\substack{DENOMINATOR \\ ZERO}}$ contained within the circle in the xy momentum space plane with elastic scattering momentum radius P_{IF}.

$$n_p = \frac{b^2 P_{IF}^2}{4\pi\hbar^2} \tag{15}$$

The total cross section, σ_{total} is then given approximately by the product of (15) and (13) as

$$\sigma_{TOTAL} \simeq \frac{|P_{Io}|^2 B^2 N}{4\hbar^4 c^2} \tag{16}$$

(16) is proportional to N in consequence of the fact that the peak values (10) in the differential cross section are multiplied by a solid angle for each peak, inversely proportional to N. A similar result is obtained for one and three dimensional scatterer arrays.

5

Expressions (9), (13) are given in the literature and describe the scattering[4] of x rays very well.

A Method for Obtaining Cross Sections Proportional to N^2

In order to obtain a total cross section proportional to N^2, a method is required which does not lead to the very small solid angles (12).

Each scatterer should be represented by a wavefunction and exchange of momentum with the scatterer must be taken into account. If a scatterer exchanges momentum $\Delta \bar{p}$, the expectation value of its momentum after scattering is altered by $\Delta \bar{p}$. This requires the scatterer wavefunction ψ_{SF} after scattering to be related to the wavefunction ψ_{SO} before scattering by

$$\psi_{SF} = \psi_{SO} \, e^{i \Delta \bar{p} \cdot \bar{n}/\hbar} \qquad (17)$$

For such exchange by the nth scatterer, the integrand of (2) would therefore contain a factor

$$e^{i(\bar{p}_{IO} - \bar{p}_{IF} - \Delta \bar{p}) \cdot \bar{n}_n/\hbar} \qquad (18)$$

(18) suggests that the solid angle (12) will be modified. To explore this possibility and for later applications we employ the relativistic quantum mechanics.

Interaction of Four Current Densities

Let us consider the S matrix for interaction of two four current densities[5,6] given by

$$S = \frac{1}{\hbar c} \int \langle F | \overline{\Psi}_S \Gamma \Psi_S \overline{\Psi}_I \Xi \Psi_I | 0 \rangle \, d^4 x \qquad (19)$$

$|F\rangle$ is the final state, $|0\rangle$ is the original state. $\overline{\Psi}_S$ is a creation operator for scatterer S, $\overline{\Psi}_I$ is a creation operator for incident particle I. Ψ_S and Ψ_I are the corresponding annihilation operators. Γ and Ξ are position independent operators.

The operators $\overline{\Psi}_S$ and $\overline{\Psi}_I$ are represented[*] by the following ansions:

$$\overline{\Psi}_S = \sum_n \sum_j \Psi^*_{Sjn}(\bar{n} - \bar{n}_n) a^\dagger_{jn} \qquad .20)$$

$$\overline{\Psi}_I = \frac{1}{\sqrt{V}} \sum_k \overline{U}_{Ik} e^{-\frac{i}{\hbar} \bar{P}_{Ik} \cdot \bar{n}} d^\dagger_k \qquad (21)$$

Here $\bar{n}$ is again the position three vector, $a^\dagger_{jn}$ is a creation operator for the state with wavefunction Ψ^*_{Sjn}; as before n refers to the nth scattering site. $d^\dagger_k$ is a creation operator for an incident particle with known momentum $\bar{P}_{Ik}$. U_{Ik} is an incident particle spinor.

We now consider N scatterers in a solid. For the states Ψ_{Sjn}, harmonic oscillator states are selected. For a harmonic oscillator wavefunction

---

*This follows reference 5 page 219

centered at radius vector $\vec{\pi}_n$

$$\psi_{son} = K^{3/2} \pi^{-3/4} e^{-\frac{K^2}{2}|\vec{\pi}-\vec{\pi}_n|^2} \qquad (22)$$

In (22) K specifies the volume of each scatterer. For the N scatterers, the original state is taken to be

$$a_{o1}^{T} a_{o2}^{\dagger} a_{o3}^{T} \dots a_{oN}^{\dagger} \left| \begin{matrix} VACUUM \\ STATE \end{matrix} \right\rangle \qquad (23)$$

For nuclei in a solid, the wavefunctions of different scatterers will not overlap to a significant degree, and the symmetry of the many particle wavefunction need not be considered.

Let us assume now that the scatterer position probability distribution $\psi_{sJn}^{*} \psi_{sJn}$ is not changed by the scattering,

$$\left(\psi_{sJn}^{*} \psi_{sJn} \right)_{\substack{BEFORE \\ SCATTERING}} = \left(\psi_{sJn}^{*} \psi_{sJn} \right)_{\substack{AFTER \\ SCATTERING}} \qquad (24)$$

(24) implies that each final scatterer state $\left(\psi_{sJn} \right)_{F}$ may be related to the original state by

$$\left(\psi_{sJn} \right)_{F} = \left(\psi_{sJn} \right)_{o} e^{i(\Delta p_r)_n x^r / \hbar} \qquad (25)$$

(25) implies that each component in the momentum decomposition of the nth scatterer is shifted by the momentum $(\Delta p_r)_n$, corresponding to momentum exchange Δp_r.

8

Suppose there is exchange of momentum $(\Delta p_r)_n$ at the nth site, from (25) $\overline{\psi}_s$ in (19) must be replaced by

$$\overline{\psi}'_s = \sum_{n}' \psi^*_{son} a^\dagger_{on} e^{-i(\Delta p_r)_n x^r/\hbar} \tag{26}$$

Expressions 20 - 26 are employed to evaluate the S matrix (19), for initial and final scatterer states which are harmonic oscillator ground states. Let us now consider the case of spin zero scatterers, $\Gamma = 1$.

$$S = \frac{\overline{U}_{If} \Xi\, U_{Io}}{\hbar c V} \int \sum_{n=1}^{n=N} K^3 \pi^{-3/2} e^{-K^2|\vec{n}-\vec{n}_n|^2 + \frac{i}{\hbar}(p_{Io} - p_{IF} - \Delta p_n)_r x^r} d^4x \tag{27}$$

Scattering Cross Sections

Suppose now that we have scatterers in a cubic crystal with N identical cells, each with length b. For these assumptions the S matrix (27) is integrated over the crystal volume, and over the time interval $-\frac{\tau}{2} \tau_o +\frac{\tau}{2}$. τ is a time long compared with any relevant energy level periods. The result is

$$S = \overline{U}_{IF} \Xi\, U_{Io} X Y Z T \left(\frac{1}{\hbar V}\right) \tag{28}$$

9

with

$$X = \sum_{n=1}^{n=N^{1/3}} e^{\frac{i}{\hbar}(p_{Io} - p_{If} - \Delta p_n)_x X_n} - \frac{1}{K^2}\left(\frac{p_{Io} - p_{If} - \Delta p}{2\hbar}\right)^2_x \quad (29)$$

In (29) X_n = nb, with corresponding definitions for Y and Z.

$$T = \frac{\sin\left[\frac{(E_{If} - E_{Io} + E_{Sf} - E_{So})\tau}{2\hbar}\right]}{\left[\frac{E_{If} - E_{Io} + E_{Sf} - E_{So}}{2\hbar}\right]} \quad (30)$$

E_{IF} and E_{SF} are the final state energies of the incident particle and ensemble of scatterers respectively, E_{IO} and E_{SO} are the corresponding original energies.

The scattering cross section is given by * σ , with

$$\sigma = \sum \frac{V(S-1)^2}{c\tau} = \frac{V}{(2\pi)^6 c\tau\hbar^8}\int |\bar{U}_{If}^* U_{Io} XYZT|^2 d\bar{p}_S d\bar{p}_I \quad (31)$$

In (31) $d\bar{p}_S$ is the element of momentum space for the final state of the ensemble of scatterers, $d\bar{p}_I$ is the element of momentum space for the final state of the incident particle. T in (28) - (31) is a function of

_____

*This follows the procedure given in reference 7 chapter 3

the momentum variables in X, Y and Z. The integration (12) is carried out in the following way:

The length L of the crystal is given by $L = N^{1/3} b$; to evaluate (31) we must make an assumption regarding $\Delta \bar{p}_n$. Let us assume that each scatterer exchanges an equal amount of momentum $\Delta \bar{p}_\alpha$. P_s is therefore a function of Δp_α. This gives for certain integrals the approximate value

$$\frac{L}{2\pi \hbar} \int X^2 dp_{sx} = \frac{L}{2\pi \hbar} \int \left\{ \left[\frac{\sin\left[\frac{N^{1/3} b (\bar{p}_{Io} - \bar{p}_{IF} - \Delta \bar{p}_\alpha)_x}{2\hbar} \right]}{\sin\left[\frac{b (\bar{p}_{Io} - \bar{p}_{IF} - \Delta \bar{p}_\alpha)_x}{2\hbar} \right]} \right]^2 e^{-\frac{2}{K^2}\left(\frac{\bar{p}_{Io} - \bar{p}_{IF} - \Delta \bar{p}_\alpha}{2\hbar} \right)^4} \right\} dp_{sx} = N^{2/3} \tag{32}$$

The integration (32) is exact in the limit $K \to \infty$ and an excellent approximation for expected values of $K \to 10^8$. Integrations over $\bar{p}_{sy}$ and $\bar{p}_{sz}$ give similar results.
Combining (31) and (32) then gives

$$\sigma = \frac{N^2}{(2\pi)^3 c \hbar^5 \tau} \int (\bar{U}_{IF} \Xi \, U_{Io} T)^2 d\bar{p}_I = \frac{N^2}{(2\pi)^3 c \hbar^5 \tau} \int (\bar{U}_{IF} \Xi \, U_{Io} T p_I)^2 \frac{d|p_I|}{dE} dE d\Omega_I \tag{33}$$

with $E = E_I + E_s$, $d\Omega_I$ is the element of solid angle into which the incident particle is scattered.

In the center of mass system[7]

$$\frac{d|p_I|}{dE} = \frac{E_{IF} \, E_{sF}}{c^2 p_I (E_{IF} + E_{sF})} \tag{34}$$

11

(34) is integrated over E first

$$\sigma = \frac{N^2}{4\pi^2 c^3 \hbar^4} \int \frac{(\bar{U}_{IF} \Xi \, U_{Io})^2 \, p_I \, E_{IF} \, E_{SF} \, d\Omega_I}{(E_{IF} + E_{SF})} \tag{35}$$

Suppose that the incident beam of particles is again in the Z direction. The solid angle associated with the forward peak is given by the first zeros

of $\sin^4\left[\frac{1}{2} N^{1/3} b \,(\bar{p}_{Io} - \bar{p}_{IF} - \Delta \bar{p}_\alpha)_x / \hbar\right]$ and $\sin^2\left[\frac{1}{2} N^{1/3} b \,(\bar{p}_{Io} - \bar{p}_{IF} - \Delta \bar{p}_\alpha)_Y / \hbar\right]$,

These give $\frac{1}{2} N^{1/3} b \,(\bar{p}_{Io} - \bar{p}_{IF} - \Delta \bar{p}_\alpha)_x / \hbar = \pi$ and $\frac{1}{2} N^{1/3} b \,(\bar{p}_{Io} - \bar{p}_{IF} - \Delta \bar{p}_\alpha)_Y / \hbar = \pi$

and $|\bar{p}_{Io} - \bar{p}_{IF}|^2 = \left(\frac{2\pi \hbar}{N^{1/3} b} + \Delta p_{\alpha x}\right)^2 + \left(\frac{2\pi \hbar}{N^{1/3} b} + \Delta p_{\alpha Y}\right)^2$, therefore

$$d\Omega \approx \pi |\bar{p}_{Io} - \bar{p}_{IF}|^2 / |p_{Io}|^2 = \left[\left(\frac{2\pi \hbar}{N^{1/3} b} + \Delta p_{\alpha x}\right)^2 + \left(\frac{2\pi \hbar}{N^{1/3} b} + \Delta p_{\alpha Y}\right)^2\right] / |p_{Io}|^2 \tag{36}$$

Since momentum is conserved, and each scatterer was assumed to transfer equal momentum in a single scattering, it follows that

$$N \Delta \bar{p}_\alpha = \bar{p}_{Io} - \bar{p}_{IF} \tag{37}$$

For large N, $\Delta \bar{p}_\alpha$ is very small and the term $\frac{2\pi \hbar}{N^{1/3} b}$ in (36) will be much larger. The solid angle implied by (36) will therefore be very small and the total cross section (35) will be very small.

Momentum Exchange Possibilities

Any number of scatterers may exchange momentum in a scattering process. The total cross section must consider all possibilities. If the scatterers

12

are electrons, as in the case of x rays, each scatterer is usually bound
to a particular site and the coupling of electrons on different sites with
each other is small. Under these conditions, each electron may be expected
to exchange any amount of momentum. If such exchange is a random process
each electron would exchange approximately $\Delta \bar{p}_B$

with

$$\Delta \bar{p}_B \simeq \frac{\bar{p}_{Io} - \bar{p}_{If}}{\sqrt{N}} \tag{38}$$

For large N, (38) is so small that the momentum transfer does not play
a significant role · The total cross section has the very small value
implied by the small solid angle into which an incident particle is
scattered.

Suppose that the nuclei of a solid are the scatterers. These may be
very tightly coupled to each other. If the incident particles have very
low energy, the following process may occur. All of the momentum
may be exchanged at a single nucleus. The tight binding* of that nucleus
to other nuclei would result in the momentum being quickly transferred to
the entire lattice.

For exchange of momentum at a single scatterer at site $\bar{\pi}_n$, (26)
will be replaced by

$$\bar{\psi}'_{S_n} = \psi^*_{So_n} a^+_{o_n} e^{-\iota \Delta p_r x'/\hbar} + \sum_{i \neq n} \psi^*_{So_i} a^+_{o_i} \tag{26A}$$

13

*Tight binding implies scatterer quantum states with well defined positions. In appendix A it is shown that scatterer states of well defined momenta give small total cross sections.

(26A) may be written in a more illuminating form by adding $\Psi^*_{son} a^\dagger_{on}$ to the last term and subtracting it from the first term to give

$$\bar{\Psi}'_{S_n} = \Psi^*_{son} a^\dagger_{on} \left[e^{-i\Delta p_r x^r/\hbar} - 1 \right] + \sum_{ALL\, n} \Psi^*_{soi} a^\dagger_{oi} \qquad (26B)$$

In (26B) the last term is the probability amplitude for the possible process where no momentum is exchanged at any site. The first term then represents the contribution to the amplitude for exchange of momentum at the nth site. We assume strong coupling of nuclei to each other with no possible way of identifying the scattering site. Therefore, we must sum only the first term in (26B) over all possible sites. Carrying out this sum then gives

$$\bar{\Psi}''_S = \sum_j \Psi^*_{soj} a^\dagger_{oj} e^{-i\Delta p_r x^r/\hbar} \qquad (26C)$$

(26C) gives a solid angle

$$d\Omega \approx \pi \left[\left(\Delta p_x + \frac{i\hbar\pi}{N^{1/3}b} \right)^2 + \left(\Delta p_Y + \frac{2\hbar\pi}{N^{1/3}b} \right)^2 \right] \Big/ |p_{Io}|^2 \qquad (39)$$

If $\bar{p}_{Io}$ is sufficiently small, a total momentum transfer with $\Delta\bar{p} \rightarrow 2\bar{p}_{Io}$

15

is possible without the momentum transfer changing the coupled scatterer wavefunction enough to permit identifying that scatterer after scattering.

Under these conditions (39) may approach 4π and (35) may approach the value

$$\sigma \approx \frac{|U_{IF} \equiv U_{Io}|^2 \, E_{IF}^2 \, N^2}{\pi \, \hbar^4 c^4} \tag{40}$$

The large cross section (40) implies that the kinematics of the exchange does not restrict the value of the solid angle into which an incident particle is scattered. In Appendix B it is shown that this is indeed the case.

We may also imagine processes in which two, three, or any number of unidentified scatterers exchange all of the momentum. In Appendix C these possibilities are considered and it is shown that the single unidentifiable scatterer case gives the largest cross section.

Limits of Validity of the Formula for the Total Cross Section

A crystal would have to be infinitely stiff for every incident particle to be scattered with the large cross section (40).

Available crystals might be expected to have cross sections approaching (40) if: a) the energy of interaction of an incident particle with a scatterer is small compared with the binding energy of each scatterer to other scatterers. b) The recoil energy of each scatterer is small compared with the "Debye" temperature energy kT_{DEBYE}. This follows from the theory[8,9,10] of the

Mossbauer effect. This theory gives the fraction of gamma ray emissions which result in recoil of the crystal as a whole, and the fraction which result in recoil of the emitting nucleus exciting lattice vibrations. Clearly the recoil of the crystal as a whole corresponds to the infinite stiffness case discussed here. The same theory must apply for momentum transfer by an incident scatterer.

At temperature T small compared with the Debye temperature T_{DEBYE}, the fraction of Mossbauer gamma ray emissions which results in recoil of the entire crystal is calculated to be f with[10]

$$f = e^{-\frac{E_R}{kT_{DEBYE}} \left(\frac{3}{2} + \frac{\pi^2 T^2}{T^2_{DEBYE}} \right)} \tag{41}$$

In (41) E_R is the recoil energy given in terms of the individual scatterer mass μ_s by $\frac{(\Delta p)^2}{2\mu_s}$.

If (41) approaches unity this is clearly sufficient to guarantee a very large total cross section. It is not certain that this is necessary.

In the Mossbauer effect, the narrow line widths are associated with the recoil of the entire crystal with no phonon excitation. If phonons are excited, each gamma ray would have energy shared with a given type of phonon excitation. Since there are many ways of exciting the lattice this will give a larger line breadth than excitation of no phonons.

For the single scatterer momentum exchange discussed here, it is only necessary that after scattering, the single scatterer wavefunction should not be changed so much that its identity may be established by subsequent

measurements. It remains to be proved that this can or cannot be done if phonons are excited.

Coherent Scattering of Neutrinos and Antineutrinos

Let us apply (40) to the scattering of neutrinos and antineutrinos. The neutral current interaction then gives

$$\sigma = \frac{G_\omega^2 N^2}{4\pi^2 \hbar^4 c^4} \int E_\nu^2 \left\langle \left(\bar{U}_{S_0} \Gamma^\alpha (1+\Gamma_5) U_{\nu_0} \bar{U}_{\nu_F} \Gamma_\alpha (1+\Gamma_5) U_{S_0} \right|^2 \right\rangle d\Omega_\nu \tag{42}$$

It is possible to show that [11]

$$\bar{U}_{S_0} \Gamma^\alpha (1+\Gamma_5) U_{\nu_0} U_{\nu_F} \Gamma_\alpha (1+\Gamma_5) U_{S_0} = \bar{U}_{S_\eta} \Gamma^\alpha (1+\Gamma_5) U_{S_0} \bar{U}_{\nu_F} \Gamma_\alpha (1+\Gamma_5) U_{\nu_0} \tag{43}$$

In "spinor" representation

$$\Gamma_5 = \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} \qquad\qquad \bar{\Gamma} = \begin{vmatrix} 0 & -\bar{\sigma} \\ \bar{\sigma} & 0 \end{vmatrix} \tag{44}$$

All elements here are 2 x 2 matrices

$$\bar{\Gamma}(1+\Gamma_5) = \begin{vmatrix} 0 & -2\bar{\sigma} \\ 0 & 0 \end{vmatrix} \tag{45}$$

18

Let

$$U_s = \begin{vmatrix} n_s \\ \chi_s \end{vmatrix} \qquad\qquad U_2 = \begin{vmatrix} n_2 \\ \chi_2 \end{vmatrix} \qquad\qquad (46)$$

n and X are 2 component spinors

$$\bar{U}_s F(1+r_5)U_s = -2\chi_s^* \bar{\sigma}\chi_s \qquad\qquad (47)$$

$$\bar{U}_s r^0(1+r_5)U_s = 2\chi_s^* \chi_s \qquad\qquad (48)$$

therefore

$$\bar{U}_s r^\alpha(1+r_5)U_2 \bar{U}_2 r_\alpha(1+r_5)U_s = \frac{4G_w}{\sqrt{2}}\left[\chi_{sF}^* \chi_{so}\chi_{2F}^* \chi_{2o} - \chi_{sF}^* \bar{\sigma}\chi_{so}\cdot\chi_{2F}^* \bar{\sigma}\chi_{2o}\right] \qquad (49)$$

For unpolarized scatterers, the last (spin terms) in (49) average to zero.

Suppose the incident direction is again the z direction. For scattering through an angle θ , the spinor transformation law leads to

$$\chi_{2F}' \chi_{2o} = \cos\frac{\theta}{2} \qquad\qquad (50)$$

Integration of (42) then gives for the total cross section [*]

$$\sigma = \frac{4 G_w^2 E_\nu^2 N^2}{\pi \hbar^4 c^4}$$

(51)

Experiments

Experiments have been carried out to directly observe the coherent

scattering. A titanium tritide source was employed in a stainless steel

[*]
(51) is the total cross section for N identical scatterers. Required

modifications for Quark models will be considered in another paper.

---

*(51) is the same for both neutrinos and antineutrinos. In general, if all
terms in 49 contribute significantly, the neutrino and antineutrino cases
would not be identical.

thin walled container. The target was a single crystal of sapphire 2.5 centimeters in diameter and 0.38 cms thick.

In (41), $f \rightarrow 1$. The antineutrinos have an average energy about 12 kilovolts and the Debye temperature of sapphire is approximately 1000 Kelvin.

The torsion balance shown in Figure 2 was employed, for measurement of the momentum transferred to the crystal by antineutrinos.

A closed loop servosystem was developed to measure the forces exerted on the crystals. A radio frequency bridge became unbalanced whenever the torsion balance was displaced from its equilibrium position. The unbalance voltage was amplified, and then employed to produce an electrostatic force to restore the balance to its equilibrium position. The force is measured by observing the output voltage.

The torsion balance was enclosed by a hollow cylinder 20 cm in diameter, about 50 cm high, and maintained at a pressure of 10^{-6} torr by a Vac Ion pumping system.

Two small diameter aluminum cylinders extended from the top of the apparatus into the region close to the target crystals. The outside diameter of the cylinders was 16 mm and the wall thickness was 1.5 mm. The inner region of the cylinders was open to the atmosphere and the lower end was sealed with a welded disk. A motor driven gear train was employed to raise and lower a titanium tritide antineutrino source in one cylinder and an identical untritiated "dummy" in the second cylinder.

A 3,000 Curie tritium source generates 0.1 watts as a result of the kinetic energy of the 6 kilovolt Beta decay electrons. A torsion balance will respond to a heat source for several reasons. Thermal gradients may result in unbalanced gas pressures, thermoelectric effects may generate potentials.
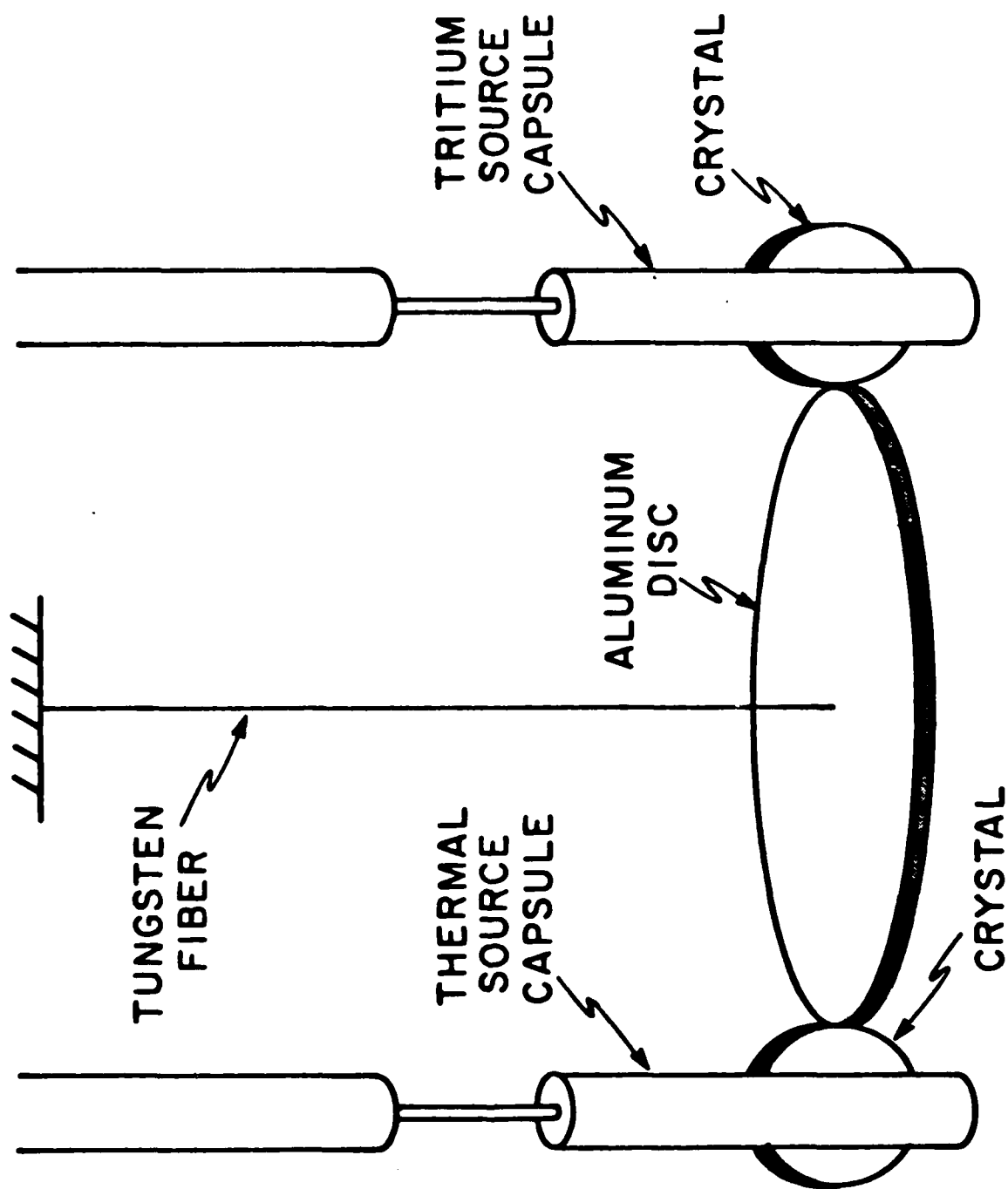
21

TRITIUM SOURCE CAPSULE

CRYSTAL

ALUMINUM DISC

TUNGSTEN FIBER

THERMAL SOURCE CAPSULE

CRYSTAL

FIGURE 2

22

In order to make the thermal effects as small as possible, the small cylinders
in which the tritium source moved, were enclosed in several layers of super-
insulation. It was observed that the thermal effects had a relaxation time
of several minutes. To reduce the thermal contribution, the servosystem was
designed with a response time of nine seconds. It was also observed that
the sign of the thermal response was opposite in the two cylinders employed
for raising and lowering the tritium source and the "dummy." An electrically
heated resistance was installed in the "dummy." Observations then indicated
that the thermal response force was less than ten percent of antineutrino
forces observed in these experiments. Since the masses of the tritium source
and "dummy" capsules were equal, the gravitational force of the tritium source
capsule was also balanced out.

Absolute Calibration of the Torsion Balance

A knowledge of all electrode dimensions and spacings makes it possible
to interpret the servosystem output voltage in terms of the forces. A more
precise procedure is to remove the tritium source and "dummy," and substitute
a lead mass in one side of the balance. The gravitational interaction of the
lead mass and target crystal provides a known force, to calibrate the torsion
balance in terms of the servosystem output voltage.

Response of the Torsion Balance to Applied Forces

The torsion balance is a damped harmonic oscillator. Periodic processes
associated with raising and lowering of a lead calibration mass or the anti-
neutrino source produce an oscillator response which may be calculated. For
a gravitational interaction the Greens solution giving displacement $x_g(t)$ at

time t for oscillator initial displacement and oscillator velocity zero at some earlier time t_e is

$$X_g(t) = -K_m' \int_{t_e}^{t} \frac{e^{-\alpha(t-t')} \sin \omega_0(t-t') \, dt'}{\left(a^2 + \nu^2(t'-t_0)^2\right)^{3/2}}$$ (52)

In (52) K_m' is a constant, α is the oscillator damping constant, ω_0 is the normal mode angular frequency, a is the distance of closest approach of the source to one of the masses, v is the velocity of the source, t_0 is the time at which the source reaches the closest approach point and is brought to rest. v is a function of time. The factor 3/2 accounts for the dependence of interaction on distance and for the component of force which couples to the oscillator. The neutrino source and dummy are expected to exert a net force with opposite sign, dependent on the solid angle of the target. The oscillator displacement is, therefore, for antineutrinos

$$X_\nu(t) = K_\nu' \int_{t_e}^{t} \frac{e^{-\alpha(t-t')} \sin \omega_0(t-t') \, dt'}{\left(a^2 + \nu^2(t'-t_0)^2\right)^{2}}$$ (53)

Operation of the Experiments

Cycles of applied forces were carried out in the following way.
At t = o a pulse is written on magnetic tape and a motor is switched on.
The motor lowers the source to a point about 22 millimeters from the center
of the target mass in 11 seconds. The source is at rest at this closest
approach point for 21 seconds. At 32 seconds another pulse is written on
the magnetic tape. This starts the motor to raise the source in 11 seconds.
The source is at rest at its most distant interaction point from 43 seconds
to 64 seconds. At 64 seconds the cycle is repeated.

Figure 3 shows the computer drawn magnetic tape recorded torsion
balance output voltage for the 27 gram lead calibration mass, averaging
over 3013 cycles with 64 second period. Figure 4 shows the recorded output
for the antineutrino source and dummy, averaging over 9826 cycles with 64
second period.

Figure 5 shows the predicted output waveform for 64 second cycles for
the gravitational interaction described by the Greens function of expression
(52). Figure 6 shows the predicted output waveform for 64 second cycles for
the antineutrino interaction described by the Greens function of expression 53.

A study of Figures 3 and 4 indicates that the forces associated with
the antineutrino source are repulsive as expected for a scattering experiment.
Figures 3, 4, 5 and 6 indicate that the gravitational and antineutrino forces
observed with the torsion balance are in good agreement with predictions.
It should be stressed that the observed data of Figures 3 and 4 include
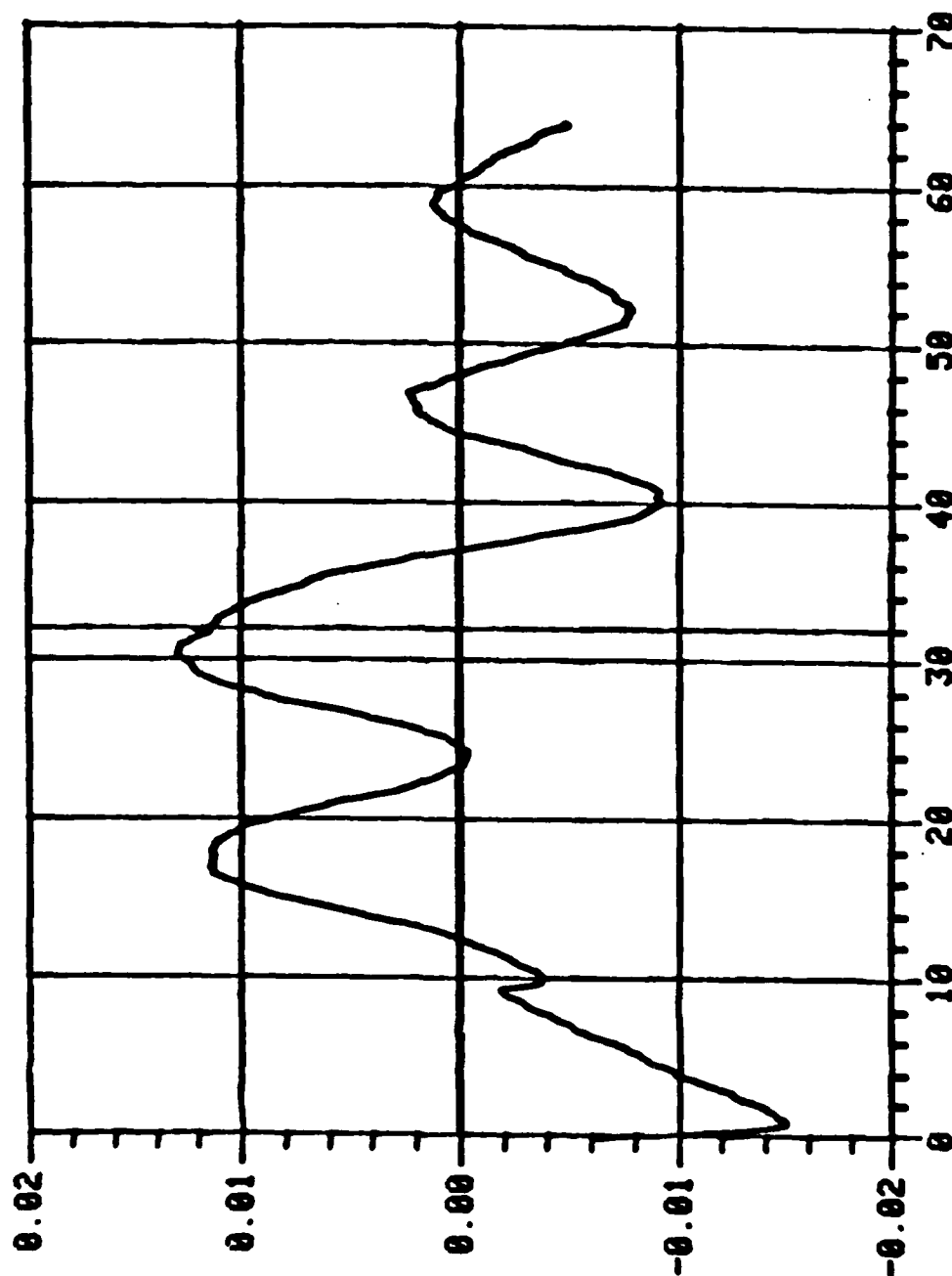effects of large amounts of ground noise.

AVERAGED SIGNAL: 214 11: 3: 7.2   'C'

FIRST HALF MEAN:-.03143   SECOND HALF MEAN: .03143
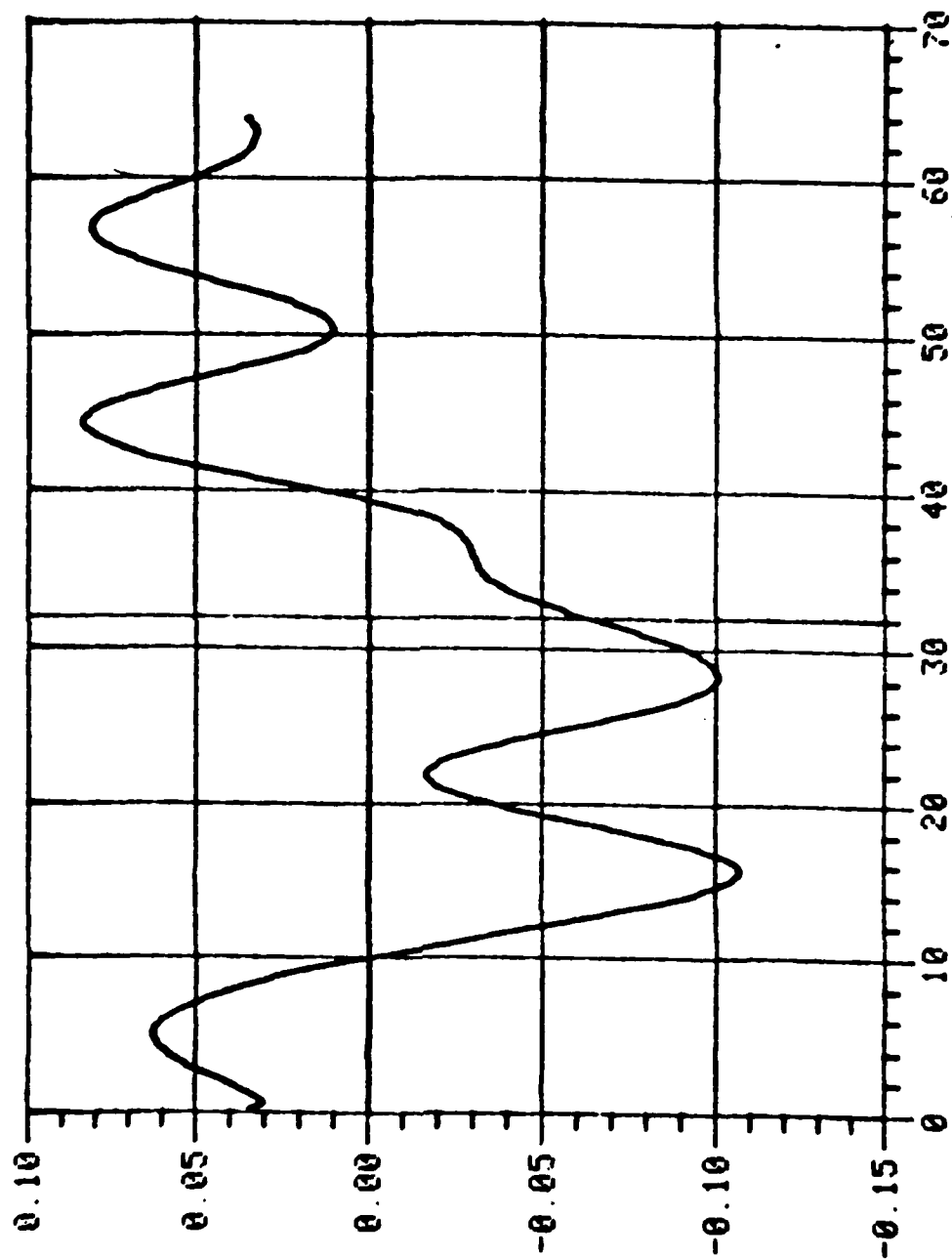
VOLTS VS. TIME FOR 3013 CYCLES.
DRIFT CORRECTION: .00000   SCALE:1.00000
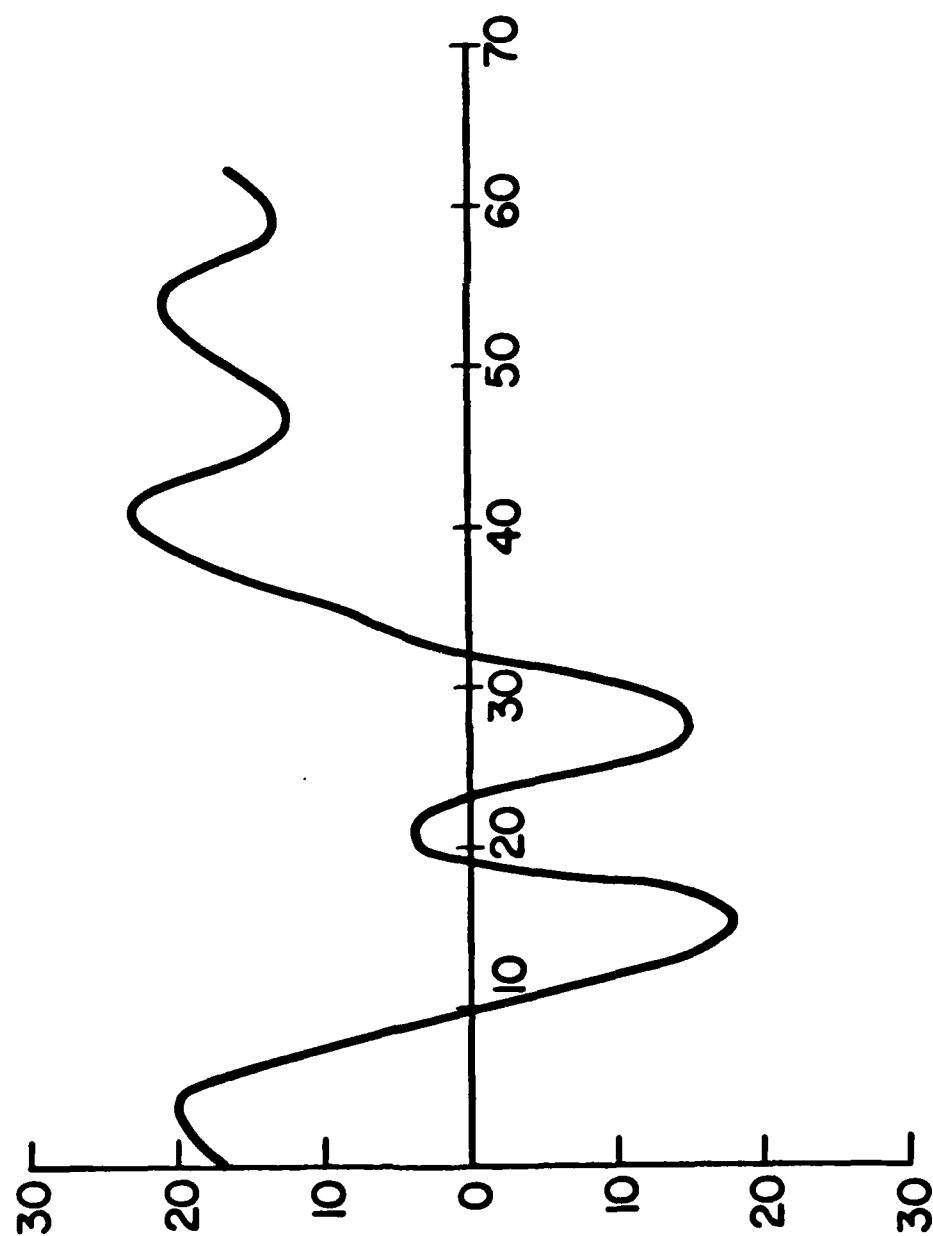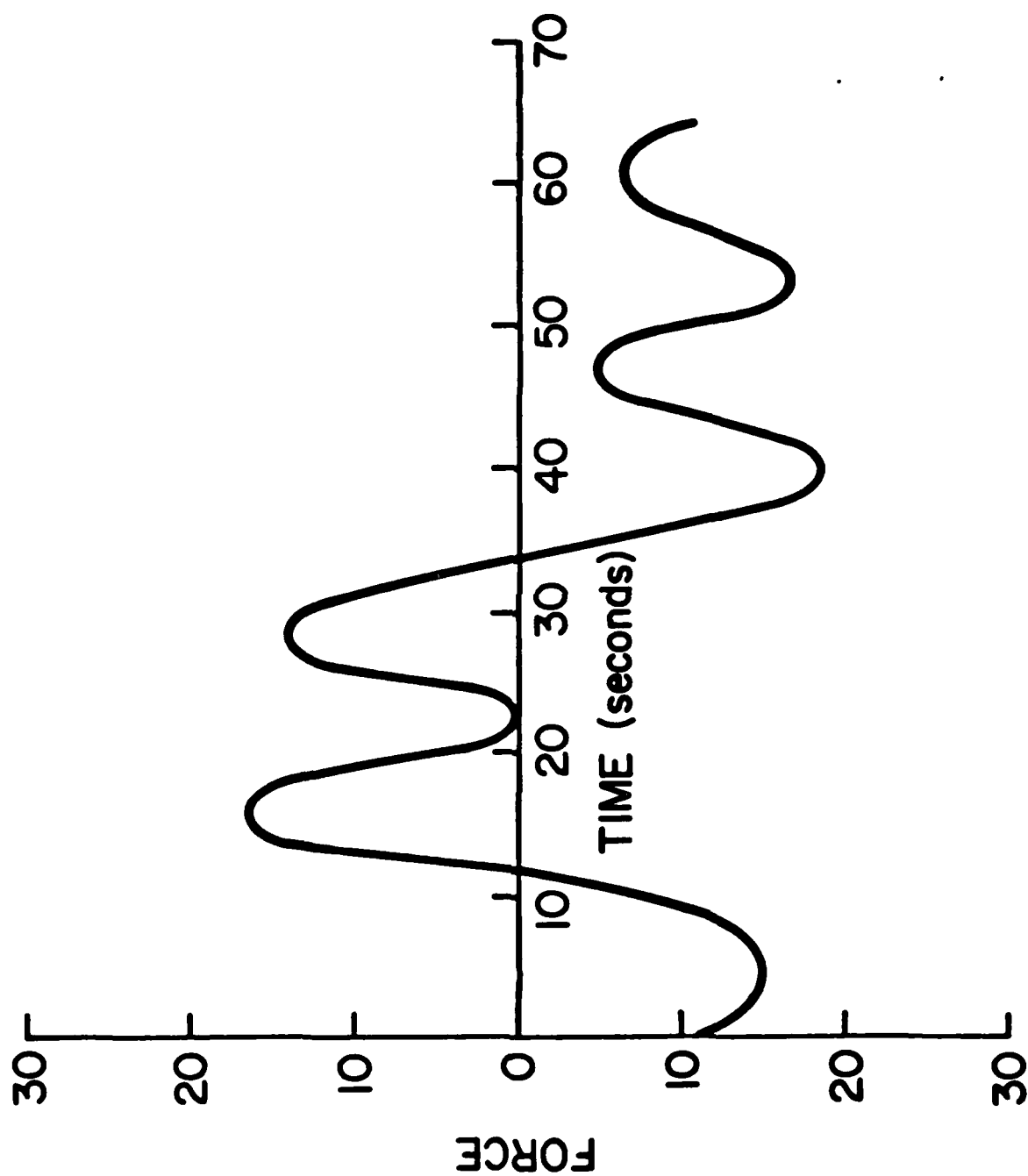
FIGURE 3

26

FIGURE 4

27

FIGURE 5

28

FIGURE 6

29

Results

Calorimetric measurements gave a value 775 ± 200 Curies for the neutrino source, in October 1983. The Oak Ridge National Laboratory had prepared a 3000 Curie titanium tritide source six months earlier. This consists of a tritiated titanium sponge inside a stainless steel cylinder one cm in diameter with wall thickness 0.8 mm. The stainless steel plus the aluminum cylinder walls are known to be sufficient to bring all Beta decay electrons to rest. These have kinetic energy less than 20 kilovolts. Measurements indicated that there was no significant gamma ray output.

The half life of tritium is about 12 years. The observed loss of activity on a time scale of months may be due to destruction of the titanium tritide chemical bonds as a result of bombardment by the Beta decay electrons. If we assume an exponential decay law, the activity during the periods associated with Figure 4 is believed to be 1340 ± 350 Curies.

Let $\langle |p_{\nu t}| \rangle$ be the average momentum transferred by an antineutrino to the crystal, and let $\langle |p_\nu| \rangle$ be the average magnitude of the antineutrino momentum. (50) implies that the differential cross section is proportional to $\cos^2\left(\frac{\theta}{2}\right)$, therefore

$$\langle |p_{\nu t}| \rangle = \frac{\int \langle |p_\nu| \rangle (1-\cos\theta) \cos^2\frac{\theta}{2} \sin\theta \, d\theta}{\int \cos^2\frac{\theta}{2} \sin\theta \, d\theta} = \langle |p_\nu| \rangle \quad (54)$$

30

Let V_ν be the servosystem output voltage with the antineutrino source and let V_G be the servosystem output with the lead mass interacting with the crystal, in each case at the distance of closest approach. Let G be Newton's constant of gravitation, 6.67×10^{-8} dyne cm^2 gm^{-2}. Let $\bar{\phi}$ be the number of antineutrinos per second per unit solid angle. Let r_G be the distance of closest approach for the lead mass and let r be the distance of closest approach for the antineutrino source, in each case from the center of mass of the target to the axis of the cylinder into which antineutrino source and lead mass are lowered. Let m_ℓ be the mass of the lead weight and let m_c be the mass of the target crystal. The observed cross section is then

$$\sigma_{OBSERVED} \approx \frac{G_g \, m_\ell \, m_c \, V_\nu \, r_\nu^2 \, K_g}{\phi \, V_G \, r_G^2 \, \langle |p_\nu| \rangle} \tag{55}$$

$$m_\ell = 26.98 \; GM$$
$$m_c = 12.73 \; GM$$
$$r_\nu = r_G \approx 22 \, mm$$

K_g is a correction for the gravitational interaction of the lead mass with the balance mass which supports the crystal.

31

These values give for the observed cross section

$$\sigma = 1.06 \pm 0.25 \ cm^2 \tag{56}$$

(56) is based on three sets of observations, with two sources and two sets of torsion balance periods.

The crystal area is 5.1 cm^2. This suggests several possible interpretations of (56) which are being explored. One is that the crystal is imperfect. Another is that the crystal does scatter every antineutrino as predicted by formula (51). The computed cross section (51) exceeds the crystal area by about a factor 20. It may therefore be true that every antineutrino is scattered twice, on average. This assumption gives agreement of the observed force with that calculated from the known antineutrino average momentum, to about 25 percent. The total repulsive force associated with an average of 2 scatterings is in fact much smaller than for one scattering.

It is noted that this is a "blind" experiment. All data are on a magnetic tape which is processed by a programmer who has no information concerning what is expected. Figures 3 and 4 without corrections, imply that a repulsive force is being observed with magnitude consistent with a large coherent scattering cross section.

Experiments at Higher Energies

Theory and experiments presented thus far imply the very large cross sections in the limit of zero neutrino energy. Neutrinos from a fission reactor have average energy exceeding one MEV. Expression (41) predicts that for transfer of momentum corresponding to one MEV

$$f \rightarrow e^{-670} \tag{57}$$

(57) implies that a one MEV momentum exchange is most unlikely to be observed. However for a momentum exchange corresponding to 10 kilovolts, (41) gives

$$f \rightarrow e^{-.067} \tag{58}$$

(58) implies that essentially all scatterings with 10 kilovolt momentum transfer will have a large cross section. An incident one MEV antineutrino which scatters and exchanges 10 kilovolts, will continue mainly in the forward direction, Figure 7.

Figure 7

The momentum transferred to the crystal is approximately

$$P_{TRANS} = P_{\nu} (1 - \cos \theta) \tag{59}$$

33

For small θ (59) is very small. The distribution function for antineutrino energy, (41), and (59) lead to an average value of momentum exchanged given by

$$\langle p \rangle = \frac{4 G_W^2 N_s^2 (FLUX) \int_0^{E_{MAX}} E^5 [(E_0-E)^2 - M_e^2 c^4]^{1/2} (E_0-E) \cos^2 \frac{\theta}{2} (1-\cos\theta) \sin\theta \, e^{-\frac{3E^2 \sin^4(\theta/2)}{M_n^2 c^2 k T_D}} \, d\theta \, dE}{\pi \hbar^4 c^5 \int_0^{E_{MAX}} E^2 [(E_0-E)^2 - M_e^2 c^4]^{1/2} (E_0-E) \, dE}$$

(60)

In (60) E_0 is the maximum energy available in the Beta Decay, E_{MAX} is the maximum antineutrino energy available. G_W is again the weak interaction coupling constant. M_n is the mass of a nucleus in the crystal, N_s is the number of scatterers, M_e is the electron mass. C is again the speed of light.

Evaluation of (60), taking multiple scatterings into account suggests that a force would be observed at the National Bureau of Standards Reactor which is about ten percent of the force observed in the tritium experiment. Unfortunately the reactor is not a seismically isolated, quiet, or carefully temperature controlled site. Floor vibrations modulate the stress in the torsion balance support fiber. This causes the fiber to unwind giving apparent noise equivalent torques.

Fluid Supported Torsion Balance

The suggestion that the wire support for a torsion balance be replaced by masses floating in a fluid was made by Dr. J. Faller.[12] We have constructed two torsion balances based on this idea. One is now at the U.S. National Bureau of Standards.

Summary and Conclusion

During the past year the theory of coherent scattering of neutrinos and antineutrinos was further developed. Earlier experiments with a 3000 Curie tritium source were repeated. Most recent data are consistent with the earlier observations.

Two fluid supported torsion balances were developed and a second wire supported balance constructed. Equipment has been set up at the 10 megawatt National Bureau of Standards Reactor. Experiments are continuing with the 3000 Curie tritium source. Attempts are in progress to observe the solar neutrinos and to observe shielding effects of crystals.

APPENDIX A

Coherent Scattering With Well Defined Scatterer Momenta

The large cross sections may be observed only under some very restricted conditions. One such condition is that the scatterers have well defined positions. In this appendix it is demonstrated that the method will not give large cross sections if the scatterers have well defined momenta. Consider again the S matrix.

$$S = \frac{1}{\hbar c} \int \langle F | \bar{\Psi}_s \Gamma \Psi_s \bar{\Psi}_I K \Psi_I | 0 \rangle \, d^4x \tag{A1}$$

For well defined scatterer momenta, it is convenient to discuss the elastic scattering case in terms of the center of mass motion.

The following kinds of quantum states are chosen for the operators $\bar{\Psi}_s$ and $\bar{\Psi}_I$.

$$\bar{\Psi}_s = \sum \sum \Psi^*_{sj}(\bar{r}) a^\dagger_j \Psi^*_{scn}(\bar{r}_c) b^\dagger_n$$

$$\bar{\Psi}_I = \sum_k \bar{U}_{Ik} e^{\frac{-i\bar{p}_{Ik}\cdot\bar{r}}{\hbar}} d^\dagger_k / \sqrt{V} \tag{A2}$$

In (A 2) $\bar{r}$ is the position three vector, $a^\dagger_j$ is a creation operator for the state with wavefunction Ψ^*_{sj}. $b^\dagger_n$ is a creation operator for the center of mass state with wavefunction $\Psi^*_{scn}(\bar{r}_c)$. $\bar{r}_c$ is the center of mass position three vector. $d^\dagger_k$ is a creation operator for an incident particle with known momentum $\bar{p}_{Ik}$. $\bar{U}_{Ik}$ may be a scalar, tensor, or spinor required to describe the incident particles.

The wavefunctions Ψ^*_{scn} for the center of mass are then $e^{-i\bar{p}_c\cdot\bar{r}_c/\hbar}$ and the three space part of the integral (A 1) may be written as

$$S_3 \to \bar{U}_{IF} K U_{ID} \int \psi^*_{SjF}(\bar{n}) \Gamma \psi_{S\ell o}(\bar{n}) e^{\frac{i}{\hbar}[(\bar{P}_{co}-\bar{P}_{cF})\cdot\bar{n}_c + (\bar{P}_{Io}-\bar{P}_{IF})\cdot\bar{n}]} d\bar{n} \, d\bar{n}_c \qquad \text{(A3)}$$

$\bar{P}_{co}$ and $\bar{P}_{Io}$ are the original momenta of the center of mass and incident particle respectively, and $\bar{P}_{cF}, \bar{P}_{IF}$ are the final state values. Let $\bar{n}'$ be the 3 vector from the center of mass to the 3 volume element $d\bar{r}$.

$$\bar{n}_c = \bar{n} - \bar{n}' \qquad \text{(A4)}$$

Substituting (A 4) into (A 3) and carrying out the integration gives

$$S_3 \to \bar{U}_{IF} U_{ID} \delta_3(\bar{P}_{co}-\bar{P}_{cF}+\bar{P}_{Io}-\bar{P}_{IF}) \int \psi^*_{SjF}(\bar{n}') \Gamma \psi_{S\ell o}(\bar{n}') e^{-\frac{i}{\hbar}(\bar{P}_{Io}-\bar{P}_{IF})\cdot\bar{n}'} d\bar{n}' \qquad \text{(A5)}$$

The quantity $\Delta p_r = \bar{P}_{co}-\bar{P}_{cF}$ will then disappear in the subsequent integrations.

For elastic scattering the "internal" state $\psi_{Sj}(\bar{n}')$ is not changed by the scattering and $\psi^*_{SjF}(\bar{n}') = \psi^+_{S_{io}}(\bar{n}')$. In practice (A 5) will give an extremely small total cross section, because the solid angle into which scattering may occur is limited as in (12).

37

APPENDIX B

Some Kinematical Considerations for Zero Rest Mass Particles

Suppose a beam of zero rest mass particles is scattered by a large crystal with mass M, initially at rest. If momentum and energy are strictly conserved and the internal degrees of freedom of M are not excited, it may be shown that

$$\frac{1}{|P_{IF}|} - \frac{1}{|P_{IO}|} - \frac{1}{Mc}\left(1 - \sqrt{1 - \frac{P_S^2}{P_{IF}^2}\left(1 - \cos^2\varphi\right)}\right) = 0 \tag{B1}$$

$$P_{IF}^2 - P_{IO}^2 - P_S^2 + 2|P_{IO}||P_S|\cos\varphi = 0 \tag{B2}$$

In (B 1) and (B 2), following earlier definitions P_{IF}, P_{IO} refer to the final and original incident particle momenta, and P_S refers to the final momentum of the center of mass of M. φ is the angle which P_S makes with the incident particle momentum. For a given value of P_{IO} it is clear that P_{IF} and φ may have a wide range of values. An even wider range is possible in practice, since the interaction time is smaller than the length of M divided by c and the internal degrees of freedom of M may share the energy. B(1) implies, for elastic scattering, that $|P_{IF}| \stackrel{\sim}{\sim} |P_{IO}|$, and (B 2) requires either $P_S \stackrel{\sim}{\sim} 0$, or $P_S \stackrel{\sim}{\sim} 2|P_{IO}|$ cos ϕ. ϕ can therefore vary over a wide range. It follows that there are no serious restrictions on the integration (14).

APPENDIX C

Other Momentum Exchange Possibilities

Most of the present paper treats the case where a single unidentified scatterer exchanges all of the momentum. Clearly other processes might occur in which any number of unidentifiable scatterers exchange all of the momentum. All possible kinds of exchange must contribute to the total cross section. Suppose that an unidentified number of scatterers, n_s, exchange total momentum Δp_ν, not necessarily in equal fractions, so that

$$\Delta p_x = \sum_{J=1}^{J=n_s} \frac{\Delta p_x}{n_{Jx}}$$

$$\Delta p_Y = \sum_{J=1}^{J=n_s} \frac{\Delta p_Y}{n_{JY}} \qquad (C1)$$

$$\Delta p_z = \sum_{J=1}^{J=n_s} \frac{\Delta p_z}{n_{Jz}}$$

$$\Delta E = \sum_{J=1}^{J=n_s} \frac{\Delta E}{n_{JO}}$$

Corresponding to (26A) we have

$$\overline{\psi}_{sn}^* = \sum \psi_{son}^* a_{on}^\dagger \left[e^{-i\left(\frac{\Delta p_x x}{n_{Jx}} + \frac{\Delta p_Y Y}{n_{JY}} + \frac{\Delta p_z z}{n_{Jz}} + \frac{\Delta E t}{n_{Jt}} \right)/\hbar} - 1 \right]$$

$$+ \sum_N \psi_{soi}^* a_{oi}^\dagger \qquad (C2)$$

39

For construction of the S matrix each of the $N-n_s$ particles must be summed over because the scatterers cannot be identified. Care is required to sum over each particle no more than once for a particular value of n_{jx}, n_{jy}, n_{jz}. For the quantity X of equation (29) this will give

$$X = \sum_{J=1}^{J=n_s} \left[\frac{\left(1 - e^{-iN^{1/3}a\left(p_{Io}-p_{If}-\frac{\Delta p}{n_{jx}}\right)_x/\hbar}\right)}{\left(1 - e^{-ia\left(p_{Io}-p_{If}-\frac{\Delta p}{n_{jx}}\right)_x/\hbar}\right)} \right] e^{-\frac{i}{\kappa^2}\left(p_{Io}-p_{If}-\frac{\Delta p}{n_{jx}}\right)_x^2} \qquad (C3)$$

$X^2 Y^2 Z^2 T^2$ is required for the cross section. When squared the cross product terms in (C3) are expected to sum to a small value. The momentum space integrals will then consist of sums $X^2_{n_{jx}} Y^2_{n_{jy}} Z^2_{n_{jz}} T^2_{n_{jo}}$, approximately.

The boundary conditions restrict the n_{jx}, n_{jy}, n_{jz} to rational numbers exceeding 1. $\Delta p_x, \Delta p_y, \Delta p_z$ are summed over and have values determined by boundary conditions. Consideration of these requirements indicates that all possible momentum transfer combinations will be included if the n_{jx}, n_{jy}, n_{jz} have integral values from 1 to $\left(\frac{pN^{1/3}a}{2\pi\hbar}\right)$.

A given set of n_{jx}, n_{jy}, n_{jz}, will lead to a solid angle given by:

$$d\Omega = \left[\frac{\langle\Delta p_x^2\rangle}{n_{jx}^2} + \frac{\langle\Delta p_y^2\rangle}{n_{jy}^2} + \frac{\langle\Delta p_z^2\rangle}{n_{jz}^2} \right] / p_{Io}^2 \qquad (C4)$$

If $\langle\Delta p_x^2\rangle = \langle\Delta p_y^2\rangle = \langle\Delta p_z^2\rangle = p_{Io}^2/3$, then the total cross section will be given as

$$\sigma \approx \sigma_1 \left[1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots \right] = \sigma_1 \left[\frac{\pi^2}{6}\right] \qquad (C5)$$

In (C5) σ_1 is the cross section for the one particle momentum transfer process.

For large values of n_{jx}, n_{jy}, n_{jz} the quantities $\frac{\Delta p}{n_{jx}}, \frac{\Delta p}{n_{jy}}, \frac{\Delta p}{n_{jz}}$ can be neglected. The theory then becomes the well known Bragg scattering-reciprocal lattice result.

[1]
A. H. Compton and S. K. Allison, <u>X Rays in Theory and Experiment</u>, Second Edition 1935, page 189, D. Van Nostrand Company, Inc., New York.

[2]
This follows from integration of formula 14.114 page 681 of <u>Classical Electrodynamics</u> by J. D. Jackson, Second Edition 1975, John Wiley and Sons, New York, London.

[3]
This follows from integration of formula 14.115 page 681 of reference 2.

[4]
C. Kittel, <u>Introduction to Solid State Physics</u>, 5th Edition, page 69, John Wiley and Sons, Inc., 1976.

[5]
Landau and Lifshitz, <u>Quantum Mechanics - Non Relativistic Theory</u>, Addison Wesley, Volume 3, 1958, page 215.

[6]
Relativistic Quantum Theory Part 2, Chapter XV, E. M. Lifshitz and L. P. Pitaevski 1974, Pergamon Press Ltd., Particle Physics, An Introduction, Chapter VI, M. Leon, Academic Press, 1973.

[7]
Particle Physics An Introduction, M. Leon, Academic Press, 1973, Chapter 3.

[8]
W. E. Lamb, Jr., Phys. Rev. 55, 190, 1939.

[9]
R. J. Mossbauer, Z. Physik 151, 124 (1958); Naturwissenschaften 45, 538 (1958); Z. Naturforsch 14a, 211 (1959).

[10]
Gutlich, Link, Trautwein, Mossbauer Spectroscopy and Transition Metal Chemistry, Springer Verlag Berlin Heidelberg, New York, 1978, page 9.

11. Relativistic Quantum Theory, Part 1, page 88, V.B.Berestetskii, E.M.Lifshitz, and L.P.Pitaevski, 1971, Pergamon Press Ltd.

12. J. Faller Proceedings of the Second Marcel Grossmann Meeting, Trieste, Italy 1979, North Holland Publishing Company.